2012

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Choose the correct option/Answer the following : 1×3=3

(a) What is the modulus of the determinant of a unitary matrix?

(i) 1

(ii) 0

(iii) –1

(iv) None of these

(b) What is a Hermitian matrix?

(c) What is a skew-symmetric matrix?

2. Define conjugate transpose of a matrix. Show that

$$(AB)^{+} = B^{+} A^{+}$$ 1+1=2

3. Answer any *two* questions out of *(a)*, *(b)* and *(c)* :

(a) *(i)* For three Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

prove that $\sigma_i \sigma_j = i\sigma_k$, where i, j, k are cyclic permutations of indices. 3

(ii) Show that modulus of each eigenvalue of a unitary matrix is unity. 2

(b) *(i)* Verify that

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

is an orthogonal matrix. 2

(ii) Show that

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1}$$ 3

Rajput, 167

(c) What is a frame of reference? A reference frame a rotates with respect to another frame b with uniform angular velocity $\vec{\omega}$. If the position, velocity and acceleration of a particle in frame a are represented by $\vec{r}$, $\vec{V}_a$ and $\vec{f}_a$ respectively, show that the acceleration of that particle in frame b is given by $\vec{f}_b$, where

$$\vec{f}_b = \vec{f}_a + 2\vec{\omega} \times \vec{V}_a + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$ 5

4. Answer *either* (a) and (b) *or* (c) and (d) :

Either

(a) State Cayley-Hamilton theorem. Obtain the characteristic equation of a matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

and verify Cayley-Hamilton theorem. 5

(b) Find the mutually perpendicular eigenvectors of the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$ 5

Or

(c) Show that the trace of a product of two matrices is independent of the order of multiplication. Also show that eigenvalues of a Hermitian matrix are all real and its eigenvectors corresponding to two distinct eigenvalues are orthogonal. 2+3=5

(d) For the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

determine a matrix P such that $P^{-1}AP$ is a diagonal matrix. 5

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct option/Answer the following : 1×4=4

(a) Electric field vector $\vec{E}$ is

(i) rotational

(ii) irrotational

(b) What do you understand by electrical octupole?

(c) What is meant by electrical image?

(d) Define electrical susceptibility.

6. Answer the following questions : 2×3=6

(a) The electric field due to a short dipole at a point distant 1 cm from it on its perpendicular bisector is $1 \cdot 5 \times 10^{-11}$ volt/m. Find the dipole moment.

(b) Write down Poisson's equation.

(c) What is a polar molecule? Define molecular polarizability.

7. Write down the integral as well as differential form of Gauss' law. Use this law to show that the expression for field strength at a distance r due to an infinite line charge is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

where λ is linear charge density and r is the distance of the external point from the line charge. 1+1+3=5

Or

Show that the interaction energy of two dipoles of moments $\vec{p}_1$ and $\vec{p}_2$ is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}_1 \cdot \vec{p}_2}{r^3} - \frac{3}{r^5}(\vec{p}_1 \cdot \vec{r})(\vec{p}_2 \cdot \vec{r})\right]$$

where $\vec{r}$ is the radius vector joining the centres of the two dipoles. Hence derive the torque acting on any dipole due to the field of another dipole. 4+1=5

8. Answer any *two* questions out of *(a)*, *(b)*, *(c)* and *(d)* :

(a) *(i)* Show that the electric field due to an electric dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$$

where θ is the angle between $\vec{r}$ and $\vec{p}$. 5

(ii) Show that the energy density of electrostatic field in free space is given by

$$U = \frac{1}{2}\varepsilon_0 E^2$$

where the symbols have got their usual meanings. 5

(b) (i) State and prove the uniqueness theorem regarding solutions to Laplace's equation. 1+4=5

(ii) Use Laplace's equation to find potential inside spherical capacitor. 5

(c) A point charge is situated near an infinite plane earthed conductor. Apply the method of electrical image to calculate—

(i) surface charge density induced on the plane;

(ii) the force between the plane and the charge.

An electron is at a distance 10 Å from an infinite plane conductor. Calculate the force experienced by the electron and the work done in moving it to infinite distance away from the conductor. 4+3+3=10

(d) (i) A spherical cavity is cut in a dielectric medium. Show that

$$\vec{E}_{eff} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$$

where the symbols have got their usual meanings.

(ii) Deduce Clausius-Mosotti relation. Point out its limitation. 6+1=7

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