

2 0 1 3

PHYSICS

(Major)

Paper : 4.1

Full Marks : 60

Time : 2½ hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

1. Answer any *four* of the following questions :

1×4=4

- (a) Define regular singular point at $x = 0$ for a second-order differential equation.
- (b) Prove that $p_n(1) = 1$.
- (c) State the theorem of total probability.
- (d) Write the general solution of the equation

$$y^n + 2y' + 5y = 0$$

- (e) Define mutually exclusive events.
- (f) Write a second-order, second-degree differential equation.

2. Answer any *three* of the following questions :

2×3=6

- (a) Check whether Frobenius method can be applied or not to the following equation :

$$2x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + (x-5)y = 0$$

- (b) Find the degree and order of the following two equations :

(i) $u \frac{\partial u}{\partial z} \cdot \frac{\partial^3 u}{\partial y^3} + \frac{\partial^2 u}{\partial y^2} \cdot \frac{\partial^2 u}{\partial z^2} = e^z$

(ii) $\left(\frac{d^2y}{dx^2} \right)^{2/3} = \left(y + \frac{dy}{dx} \right)^{1/2}$

- (c) Find the probability of drawing 2 aces in succession from a pack of 52 cards.

- (d) If

$$\int_{-1}^{+1} p_n(x) dx = 2$$

find the value of n .

- (e) What is the value of $p_{2n+1}(0)$?

- (f) Find the expected number of heads when n coins are tossed.

3. Answer any *two* of the following questions :

5×2=10

(a) Show that the coefficient of h^n in the expression of generating function $(1-2xh+h^2)^{-1/2}$ gives the Legendre polynomial $p_n(x)$.

(b) Establish the following recurrence formula for Legendre polynomial $p_n(x)$:

$$nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$$

(c) Show that

$$\int_{-1}^{+1} P_m(x)P_n(x) dx = 0$$

where $m \neq n$.

(d) Define mean and standard deviation.

(e) If the solution $y(x)$ of Hermite's differential equation is written as

$$y(x) = \sum_{r=0}^{\infty} a_r x^{k+r}$$

then show that the allowed values of k are zero and one only.

4. Answer any *two* of the following questions :

(a) Obtain the power series solution of the Legendre equation

$$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0 \quad 10$$

- (b) Show that for Hermite polynomial $H_m(x)$ and $H_n(x)$, the following relation is established : 10

$$\int_{-\infty}^{+\infty} e^{-x^2} H_m(x) H_n(x) dx = \begin{cases} 0, & m \neq n \\ 2^n n! \sqrt{\pi}, & m = n \end{cases}$$

- (c) (i) State the theory of compound probability and prove it. 1+5=6

- (ii) Write a short note on Gaussian distribution. 4

- (d) Prove the following recurrence relations :

(i) $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$ 3

(ii) $H'_n(x) = 2xH_n(x) - H_{n+1}(x)$ 3

(iii) $kP'_n - P'_{n-1} = nP_n$ 4

GROUP—B

5. Answer any two of the following questions : $1 \times 2 = 2$

- (a) What is the difference between the statements $a = b$ and $a = = b$?

- (b) What is an algorithm?

- (c) Name any two output devices of a computer.

6. Answer any *two* of the following questions :

2×2=4

- (a) Define high-level language. What are its advantages?
- (b) How will you test the efficiency of two algorithms?
- (c) Give the syntax of output statement in language FORTRAN or C or C++.

7. Answer any *one* of the following questions :

- (a) What is a flowchart? Draw a flowchart to find the perimeter and area of a rectangle.

1+3=4

- (b) Write a program in one of the languages FORTRAN or C or C++ to generate the N natural numbers and their sum.

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8. Answer any *one* of the following questions :

- (a) With the help of block diagram, describe briefly the basic computer operation. What are the different functional units of a digital computer? Write a note on any two of them.

5+1+2+2=10

- (b) What are the different transfer statements in FORTRAN or C or C++? Write the syntax of any three of them.

4+2+2+2=10
