2013

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 30)

1. For $z = \dfrac{1+i}{(2-3i)^2}$

(a) find Re z and Im z

(b) find Mod z

(c) find arg z

(d) give the graphical representation of z.

$1 \times 4 = 4$

2. *(a)* Find the roots of $\sqrt[3]{i}$ and locate them graphically. 2

(b) Define equivalent contour. 2

Or

Find the value of $(1+i)^5$.

3. *(a)* Determine if the following functions are analytic : 4

(i) $\dfrac{1+z}{1-z}$

(ii) e^{iz}

(b) Using Cauchy's integral formula, find the value of the integral

$$I = \oint \frac{z^2}{z^2 - 1}\, dz$$

around the unit circle at *(i)* $z = 1$, *(ii)* $z = -1$. 4

Or

Find the Taylor series expansion about the origin for $f(z) = \dfrac{1}{(1-z)^m}$ and hence find the series for $\phi(z) = \dfrac{1}{1-z}$.

4. (a) Evaluate the integral

$$I = \int_0^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$$ 7

(b) For a function $f(z)$ which has a pole of order m at $z = z_0$, show that the residue of the function at that singular point is

$$a_{-1} = \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)]_{z=z_0}$$

hence find the singular points and calculate the residues for $f(z) = \frac{e^z}{(z-2)^3}$.

5+2=7

Or

State and derive the Cauchy-Riemann conditions and then use them to compute the first derivative of $f(z) = e^z$. 5+2=7

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

5. Answer the following questions : 1×4=4

(a) State Hamilton's principle.

(b) State one advantage of Lagrangian formulation over Newtonian formulation.

(c) A system of ten particles has five holonomic constants. How many generalised coordinates are required to describe the motion?

(d) What is virtual work? State the principle of virtual work.

6. *(a)* A Lagrangian is given by

$$L = \frac{1}{2}\alpha\dot{q}^2 - \frac{1}{2}\beta q^2$$

where α and β are constants. Find the Hamiltonian of the system. 2

(b) A particle moves in a circular orbit obeying inverse square law. Show that its angular momentum varies as the square root of its radius. 2

Or

What is a cyclic coordinate? Show that a cyclic coordinate in Lagrangian is also a cyclic coordinate in Hamiltonian.

7. Answer any *two* of the following questions : 4×2=8

(a) Establish d'Alembert's principle.

(b) Set up the Lagrangian of a compound pendulum and obtain its equation of motion.

(c) Deduce an expression of reduced mass of a two-body central force problem.

8. (a) (i) Set up the differential equation of the orbit of a particle under the influence of a central force $F(r)$.

(ii) Show that if the position vector of a particle is given by $r = a\sin\theta$, then $F(r) \propto \frac{1}{r^5}$. 4+3=7

(b) If the Lagrangian of a conservative system does not contain time explicitly, show that the total energy of the system is conserved. Using Lagrange's equation, show that $F_x = -\frac{\partial V}{\partial x}$. 5+2=7

Or

Define Hamiltonian of a system and then derive Hamilton's canonical equations. 2+5=7

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