2011

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

1. Show with examples that vectors can give an algebra. 1

2. *(a)* Using scalar product of vectors, show that

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$ 2

(b) Write the null vector in explicit form. 2

3. BAC rule states

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

Then show that in general

$$(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$$

Find out the condition where equality holds. 5

Or

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of the points A, B and C in space, what is the area of the triangle?

4. *(a)* What is the physical significance of grad $\vec{A}$? 2

(b) If some scalar field is given by

$$\phi(\vec{r}) = \phi(r) = r^2 = x^2 + y^2 + z^2$$

then show that $\vec{\nabla} r$ is a unit vector. 3

(c) If $\phi(x, y, z)$ is a scalar function, express $d\phi(x, y, z)$ in terms of $\vec{\nabla}\phi(x, y, z)$. Show that the unit vector $\hat{\nabla}\phi$ must be perpendicular to any $d\vec{r}$ on a surface of constant ϕ. 2+3

OR

5. *(a)* Give the diagrammatic representation of the curl of a vector around point P. What is its zero curl representation? 3

(b) *(i)* The electrostatic force acting between two point charges q and q' at a distance r apart is

$$\vec{E} = \frac{qq'}{r^2}\hat{r}_0$$

where $\hat{r}_0$ is a unit vector along $\vec{r}$. Find out curl $\vec{E}$. 5

(ii) Justify the statement that the electric lines of force cannot be closed lines. 2

GROUP—B

(Mechanics)

(*Marks* : 40)

6. *(a)* Name the fictitious force obtained in the rotating frame of reference. 1

(b) What is the difference between laboratory frame of reference and centre of mass frame of reference? 1

(c) What is meant by moment of inertia? 1

(d) Can we have equipotential surfaces of the gravitational field of a point mass? 1

(e) What is meant by acceleration due to gravity? State some methods to determine it experimentally.

(f) Why are cyclones not set up at the equator? 1

7. (a) Identify the centrifugal force in the expression of the equation of motion in a rotating frame with angular velocity ω. Justify the statement that centrifugal force is a fictitious force. 2

(b) Calculate the mass of the sun, given that the distance between the sun and the earth is $1 \cdot 49 \times 10^{13}$ cm and $G = 6 \cdot 66 \times 10^{-8}$ CGS units. 2

8. Answer any *two* questions : 5×2=10

(a) Show that whenever a body is acted upon by a number of forces such that the resultant is not zero, then the work done by the resultant force is equal to the change in the kinetic energy of the body.

(b) The position of a moving particle at an instant is given by

$$\vec{r} = \hat{i}a\cos\theta + \hat{j}a\sin\theta$$

Show that the force acting on the particle is conservative.

(c) Explain briefly how the acceleration due to gravity is determined by Kater's pendulum in the laboratory.

9. Answer any *two* questions : 10×2=20

(a) A body of mass m is dropped from rest from height h at latitude 45° in the northern hemisphere ($h \ll$ radius of the earth). Where will it land relative to a plumb bob suspended from the point of release?

(b) Show that if a heavy (moving) particle collides elastically with a lighter particle at rest, the particle (incident) can never be scattered perpendicular to the initial direction.

(c) (i) Calculate the centre of mass of a solid hemisphere.

(ii) Show that the external force acting on an extended system of particles is equal to the rate of change of momentum of its centre of mass (CM).

(d) (i) Show that the angular momentum of an extended system is

$$\vec{L} = \vec{L}_{cm} + \vec{R}_{cm} \times \vec{M}_{cm}$$

where the symbols used in the above expression carry their usual meanings.

(ii) The density of a solid sphere varies inversely with the distance from its centre. Calculate its moment of inertia about (1) any diameter and (2) tangential axis.

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