

2014

PHYSICS

( Major )

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks  
for the questions

GROUP—A

( Mathematical Methods )

( Marks : 30 )

1. Answer the following questions : 1×4=4

(a) What is the argument of  $-3i$ ?

(b) Express the number  $-\sqrt{6} - \sqrt{2}i$  in polar form.

(c) Plot the number  $e^{\left(1 + \frac{\pi}{4}i\right)}$ .

(d) Find the real part of  $\frac{1+z}{1-z}$ .

2. (a) Find and plot all the roots of  $(1-i)^{\frac{1}{4}}$ . 2

(b) Prove that,  $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$ . 2

3. (a) Check the analyticity and hence find the derivative of the function  $f(z) = \sin z$ . 4

(b) Using Cauchy's integral formula, evaluate

$$\oint_C \frac{z^2}{(z-1)^3} dz$$

where C is a circle given by  $|z|=2$ . 4

Or

Find Taylor series expansion about the origin for  $f(z) = \ln(1+z)$ .

4. (a) Define pole, simple pole, isolated singularity and essential singularity. 2

(b) Find Laurent expansion for the function

$$f(z) = \frac{\sin z}{z^4}$$

about  $z_0 = 0$  and hence classify the singularity and calculate the residue. 5

Or

Derive Cauchy's integral formulas. 7

5. Calculate the residues of  $f(z) = \frac{z^2}{(1+z^2)^2}$  and

evaluate  $\int_0^{\infty} \frac{x^2 dx}{(1+x^2)^2}$ . 3+4=7

( 3 )

GROUP—B

( Classical Mechanics )

( Marks : 30 )

6. Answer the following questions : 1×4=4

(a) What is the nature of orbit for an object moving under the influence of an inverse square law force with total energy  $E < 0$ ?

(b) A system of 5 particles has 12 equations of constraints and requires 3 generalized coordinates. Are the constraints holonomic or non-holonomic?

(c) Write down the Lagrange's equation of motion for a non-conservative system.

(d) What is the expression of Hamiltonian of a system in spherical polar coordinates?

7. (a) Show that angular momentum is a constant of central force motion. 2

(b) What are generalized forces and generalized momenta? 2

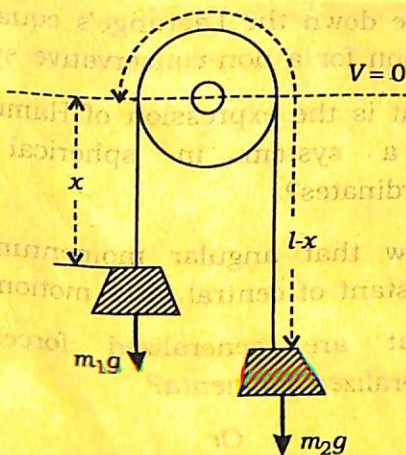
Or

Determine the Hamiltonian of a system if its Lagrangian is given by  $L = \frac{1}{5} \dot{q}^2 + \alpha(q - q_0)^2$ , where  $\alpha$  is a constant. 4

8. Answer any *three* of the following questions :

4×3=12

- (a) Obtain the general differential equation of a central orbit.
- (b) Using the variational principle, show that the shortest distance between two points in a plane is a straight line.
- (c) Referring to the figure given below, consider a system of two masses  $m_1$  and  $m_2$  tied together with a light inextensible cord of length  $l$  passing round a frictionless pulley:



Find the equation of motion for this system employing Lagrange's equation.

- (d) A simple pendulum hangs from the ceiling of an elevator which is moving down with a constant acceleration  $a$ . Obtain the Hamiltonian and hence the equation of motion of the simple pendulum.
- (e) Deduce the Hamilton's canonical equations in terms of Poisson's brackets and show that Poisson's bracket of two constants of motion is itself a constant of motion.

9. Answer any two questions :

5×2=10

- (a) A particle follows a spiral orbit given by  $r = ae^{b\theta}$  under the influence of a central force, where  $a$  and  $b$  are constants. Obtain the force law.
- (b) Show that the total energy of a particle of mass  $m$  acted upon by a central force is given by

$$E = \frac{L^2}{2m} \left[ u^2 + \left( \frac{du}{d\theta} \right)^2 \right] + V(r)$$

where  $V(r)$  is the potential energy,  $L$  the angular momentum and  $(r, \theta)$  the polar coordinates of the particle;  $u = 1/r$ .

- (c) Starting from the Hamilton's principle, deduce the Lagrange's equations of motion.
- (d) Using Hamilton's canonical equations, derive the equation of motion of a particle moving in a force field in which the potential is given by  $V = -k/r$ , where  $k$  is positive.

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