2015

PHYSICS

(Major)

Paper : 1.1

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP – A

(Mathematical Methods)

Marks : 20

1. (a) Show with convincing explanation that a scalar quantity can be represented as a vector quantity. 1+1=2

(b) Show that the vector field $\vec{A}(\vec{r}) = \vec{r} \times \vec{\nabla}\phi(r)$ is orthogonal to both $\vec{r}$ and $\vec{\nabla}\phi(r)$. 1+1=2

(c) Give the diagrammatic representation that the components of a vector become different when the frame of reference from where the vector was observed is rotated by some angle. Is there any specific angle of rotation for which the components of vector do not change? 1+1=2

(d) Formulate the simplest differential operator from the concept of gradient that will not change the vectorial and parity properties of a field on which it operates. 2

(e) Show that

$$\nabla^2 r^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) r^2 = 6 \qquad 2$$

where the symbols have the usual meaning.

2. (a) Prove that if $\vec{a}, \vec{b}$ are two proper non-collinear vectors and p, q are two scalars such that $p\vec{a} + q\vec{b} = 0$, then $p = q = 0$. 3

(b) If the resultant of two forces is equal in magnitude to one of the components and perpendicular to it in direction, find the other components. 7

Or

3. (a) If $\vec{R}(u) = x(u)\,\hat{i} + y(u)\,\hat{j} + z(u)\,\hat{k}$, where x, y and z are differentiable functions of a scalar u, prove that $\frac{d\vec{R}}{dt} = \frac{dx}{du}\hat{i} + \frac{dy}{du}\hat{j} + \frac{dz}{du}\hat{k}$ 3

(b) A particle moves so that its position vector is given by $\vec{r} = \hat{i}\cos wt + \hat{j}\sin wt$

where w is a constant. Show that

(i) the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$

(ii) the acceleration $\vec{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin

(iii) $\vec{r} \times \vec{v} =$ a constant vector. 2+2+3=7

GROUP – B

(Mechanics)

Marks : 40

4. (a) Give the characteristics of inertial forces that distinguish them from real forces. 1

(b) In what respect the gravitational force cannot be considered as an inertial force ? 1

(c) State work-energy theorem. 1

(d) What are the principal moments of inertia ? 1

(e) Why the centre of mass frame is called the zero momentum frame ? 1

(f) "When a rotating body contracts, its angular velocity increases." Give reason. 1

5\. (a) Show that the conservative force is the negative gradient of potential energy. 2

(b) Calculate the centre of mass of a semi-circular disk. 2

6\. Answer any *two* questions : $5 \times 2 = 10$

(a) Find the gravitational potential and field at an internal point of solid homogeneous sphere.

(b) Calculate the moment of inertia of a solid sphere.

(c) Describe the determination of g by Kater's pendulum.

7. Answer any *two* questions : 10×2=20

(a) Show mathematically that coriolis and centrifugal forces are produced as a result of earth's rotation. Are these forces different in magnitude at different location on the earth's surface ? Explain. 8+2=10

(b) Show that the relationship between the angular momentum relative to the C.M. frame of reference of a system of particles and the angular momentum relative to the laboratory or L-frame is given by

$$\vec{L} = \vec{L}_{CM} + \vec{r}_{CM} \times \vec{P}$$

where the symbols have the usual meaning.

The position of a moving particle is given by

$$\vec{r} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}.$$

Show that the force acting on the particle is conservative. 6+4=10

(c) Explain conservation law of angular momentum.

A rigid body of solid (spherical) symmetry is allowed to roll down an inclined plane without slipping. Show that the linear acceleration of the body is

$$\frac{g \sin \lambda}{1+\frac{k^2}{a^2}}$$

where g, is the acceleration due to gravity, k, the radius of gyration and a, the radius of the body rolling down the inclined plane making an angle λ with the horizontal.

3+7=10