Total No. of printed pages = 6

3 (Sem 4) PHY M1

2015

PHYSICS

(Major)

Theory Paper : M-4.1

Full Marks – 60

Time – 2½ hours

The figures in the margin indicate full marks for the questions.

GROUP–A

1. Answer any *four* of the following questions : 1×4=4

(a) Define ordinary point of a second order differential equation.

(b) What is the value of $P_n(t)$?

(c) Define total probability.

(d) Under what condition does the Gaussian distribution become normal distribution ?

[Turn over

(e) Give an example where Legendre polynomial is used in Physics.

(f) What is meant by mean deviation ?

2. Answer any *three* of the following questions : 2×3=6

(a) Check whether Frobenius method can be applied to the following equation or not.

$$\frac{d^2y}{dx^2} - \frac{9y}{x^3} = 0$$

(b) Prove that $P_n^m(-x) = (-1)^{n+m} P_n^m(x)$.

(c) What is the probability that the ace of spaces will be drawn from a deck of cards at least once in 104 consecutive trials ?

(d) Prove the following recurrence relation : $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.

(e) Find the degree and order of the following equation :

$$\left(\frac{d^2y}{dx^2}\right)^{2/3} = \left(y + \frac{dy}{dx}\right)^{1/2}$$

3. Answer any *two* of the following questions : 5×2=10

(a) Establish the following recurrence formula for Legendre polynomial $P_n(x)$

$$nP_n(x) = (2n-1)\,xP_{n-1}(x) - (n-1)\,P_{n-2}(x).$$ 5

(b) Find the singularity of the differential equation $(1-x^2)y'' + xy' + y = 0$ and discuss the nature of the singularity. 5

(c) Find the probability of almost 5 defective fuses to be found in a box of 200 fuses, if experience shows that 2% of such fuses are defective. 5

(d) Show that $\int_{-1}^{+1} x P_n(x) P_{n-1}(x)dx = \frac{2n}{4n^2-1}$. 5

4. Answer any *two* of the following : 10×2=20

(a) (i) Using the following definition of the Legendre polynomials $P_n(x)$,

$$(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$$

show that $|P_n \cos(\theta)| \leq$ | 6

(ii) Prove the recurrence relation $xP'_n - P'_{n-1} = nP'_n$ 4

(b) (i) Find the indicial equation of the Hermite equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2xy = 0$. 5

(ii) Show that the generating function for Hermite polynomial $H_n(x)$, for integral n, and real values of n is given by

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x).$$ 5

(c) (i) What is Gaussian distribution ? 2

(ii) Define standard deviation. 2

(iii) Prove the theory of compound probability. 5

(iv) Write one property of normal distribution. 1

(d) (i) Show that $\int_{-1}^{+1} P_n(x)\, P_m(x)\, dx = 0$

where $P_n(x)$ and $P_m(x)$ are solutions of the Legendre differential equation. 6

(ii) Show that

$H_0(x) = 1$ and $H_1(x) = 2x$. 2+2=4

GROUP – B

Answer any *two* of the following : 1×2=2

(a) What is the function of control unit ?

(b) Give the logical AND operation.

(c) What is a string ?

Answer any *two* of the following : 2×2=4

(a) Define with example an operating system.

(b) What are control statements ? Give examples.

(c) Write a syntax in C/FORTRAN. How is an array declared in a program ?