2015

PHYSICS

(Major)

Paper : 5.3

(Quantum Mechanics and Astrophysics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Write the answers to the two Groups in separate books

GROUP—A

(Quantum Mechanics)

(*Marks* : 40)

1. Answer any *four* questions as directed : 1×4=4

(a) What physical phenomenon proves the particle nature of light?

(b) A krypton atom emits a photon of orange light with wavelength $\lambda \approx 606$ nm. What is the corresponding photon energy?

(c) In Compton scattering, which one of the following is true?

(i) The wavelength of scattered light is same as the wavelength of incident light

(ii) The incident photon is absorbed by the electron

(iii) The scattered wavelength is larger than the incident wavelength

(iv) The scattered wavelength is smaller than the incident wavelength

(Choose the correct option)

(d) According to de Broglie hypothesis, the kinetic energy of a particle of mass m is

(i) $k = h^2 / 2m\lambda^2$

(ii) $k = mc^2$

(iii) $k = mc^2\left(1 - \frac{v^2}{c^2}\right)$

(iv) $k = \frac{mc^2}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$

(Choose the correct option)

(Continued)

(e) Can the particle and wave nature be simultaneously observed?

(f) An electron is confined within a region of width $1 \cdot 0 \times 10^{-10}$ m. Estimate the minimum uncertainty in the momentum of the particle.
(Given $\hbar \approx 1 \cdot 05 \times 10^{-34}$ J-sec)

2. Answer any *two* questions : 3×2=6

(a) A non-relativistic free particle of mass m has kinetic energy k. Obtain an expression for the de Broglie wavelength. What is the de Broglie wavelength for an electron having kinetic energy 800 eV? Given electron mass $m_e \approx 9 \cdot 1 \times 10^{-31}$ kg. 1½+1½=3

(b) A particle with total energy E is influenced by a potential energy $V(x)$. Show that the one-dimensional Schrödinger equation can be written in the form

$$\left[\frac{d^2}{dx^2} + k^2 - U(x)\right]\psi(x) = 0$$

where

$$k^2 = 2mE/\hbar^2 \text{ and}$$
$$U(x) = 2mV(x)/\hbar^2$$

If the wave function of a particle is $\psi(x, y, z, t) = \psi(x, y, z)e^{-iEt/\hbar}$, then show that the probability of finding the particle at the point (x, y, z) is independent of time. 2+1=3

(c) If the wave function of a particle confined in a box of length L is

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

obtain the expectation value of the position $\langle x \rangle$ of the particle. 3

(d) Two quantum particles are travelling along X-axis in opposite direction. Their wave functions are combined to produce a resultant wave $\psi(x, t) = A\cos(kx)e^{-i\omega t}$. What is the probability of current density j for this wave function? Interpret your result. 2+1=3

(Continued)

3. Answer any *two* questions : 5×2=10

(a) The photoelectric work function W for lithium is 2·3 eV. Find the threshold frequency. If the ultraviolet light of wavelength $\lambda = 3000$ Å is incident on a lithium surface, calculate the maximum kinetic energy of the photoelectrons. Briefly discuss how the wave theory of light fails to explain the photoelectric effect. 1+2+2=5

(b) Prove that the angle ϕ, at which the electron in Compton effect recoils, is related to the angle θ of scattered photon as

$$\tan\phi = \frac{\cot(\theta/2)}{1+E_0/mc^2}$$

Where $E_0 = hc/\lambda$ = energy of incident photon and m is the rest mass of the electron.

An X-ray photon with wavelength $\lambda = 1$ Å is scattered by a free electron at rest. The scattering angle is $\theta = 60°$ from the incident direction. Calculate the Compton shift $\Delta\lambda$. 3+2=5

(c) In an electron diffraction experiment, electrons are accelerated by an electric potential V. Show that the de Broglie wavelength of an electron of mass m is

$$\lambda = \frac{h}{\sqrt{2meV}}$$

where e is the electronic charge. The kinetic energy of a particle in a gas with temperature T (kelvin) is $\frac{3}{2}k_B T$. What is the de Broglie wavelength of a thermal neutron at $T = 300$ K? Here k_B is the Boltzmann constant $\approx 1{\cdot}38 \times 10^{-23}$ J/K and neutron mass is $m \approx 1{\cdot}6 \times 10^{-27}$ kg. Write down the relativistic formula for de Broglie wavelength. 2+2+1=5

4. Answer any *two* questions : 5×2=10

(a) What are the properties satisfied by a physical wave function? Normalize the wave function

$$\psi(x) = Ae^{-(\frac{\alpha}{2})x^2}$$

to unity in the domain $x \in [-\infty, \infty]$. Here α is a constant. Given $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 2+3=5

(b) A particle of mass m is confined in a one-dimensional potential box of infinite height given as

$$V(x) = \infty \text{ for } x = 0,\ a$$
$$= 0 \text{ for } 0 < x < a$$

Show that the energy of the particle in nth quantum state is

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$ 5

(c) For a linear harmonic oscillator potential $V(x) = \frac{k}{2}x^2$, show that the Schrödinger equation (time-independent) takes the form

$$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \xi^2)\,\psi(\xi) = 0$$

where

$$\lambda = 2E/\hbar\omega,\ \omega = \sqrt{k/m}$$

and $\xi^2 = \left(\frac{mk}{\hbar^2}\right)^{1/2} x^2 = \left(\frac{m\omega}{\hbar}\right)x^2$

or $\xi = \alpha x$ where $\alpha = \sqrt{m\omega/\hbar}$. 5

5. Answer any *two* questions : 5×2=10

(a) Normalize the wave function $\psi(x) = A / x^2$ between $x = 1$, $x = 3$. What is the probability of finding a particle between $x = 4$ and $x = 5$? 2+3=5

(b) What is the significance of Heisenberg's uncertainty principle? A proton is confined to a nucleus of dimension $\Delta x \approx 10^{-15}$ m. Calculate the uncertainty in its momentum. What is the minimum kinetic energy of the proton? Given proton mass $m \approx 1 \cdot 6 \times 10^{-27}$ kg. 1+2+2=5

(c) Briefly discuss the Davisson-Germer experiment and its implications. 5

GROUP—B

(Astrophysics)

(*Marks* : 20)

6. Answer any *three* from the following : 2×3=6

(a) Name two bright stars in the night sky. Show the right ascension and declination in a neat celestial diagram. 1+1=2

(b) Are the altitude and azimuth of a star same for all observers on the earth? What do you mean by ecliptic? 1+1=2

(c) The star ξ Ursae Majoris has a parallax angle of $\theta \approx 0''\cdot 127$. Calculate its distance. 2

(d) An astronomer wants to observe a star with right ascension (α) and declination (δ) as

$$(23^h 20^m 39^s + 18°08'33'')$$

Which hemisphere of the earth would be the best for observing? If the star is at the meridian, what would be the sidereal time? 1+1=2

(e) A star has apparent blue magnitude $m_B = 12\cdot 4$ and the colour index is $m_B - m_V = 0\cdot 6$. If the absolute magnitude in visual band is $M_V = 6\cdot 8$, calculate the distance to the star. 2

7. Answer any *two* of the following : 4×2=8

(a) Draw a neat H-R diagram showing the main sequence, red giant, red supergiant and the white dwarf stars. Identify the location of a hot blue and a reddish cool star in the main sequence. Show the evolutionary track of a sun-like star in the H-R diagram. 1½+1+1½=4

(b) What is the energy generation mechanism inside the main-sequence stars? Which reaction cycle dominates the energy production in massive stars with high central temperature? Discuss the proton-proton chain for the synthesis of helium. 1+1+2=4

(c) The peak wavelength emitted by a star is $\lambda_{max} \approx 4000$ Å. Calculate the surface temperature of the star. Given Wien's constant, $b \approx 0 \cdot 29$ cm-K. The luminosity L, radius R and surface temperature T of a star are related by Stefan-Boltzmann law $L = 4\pi R^2 \sigma T^4$. The corresponding quantities for the sun are $L_\odot$, $R_\odot$ and $T_\odot$.

A star's surface temperature is $\frac{1}{2}T_{\odot}$ but its luminosity is $10^4 L_{\odot}$. How much big is the star compared to the sun? Can you give example of one such star in the night sky? 1½+1½+1=4

8. Write short notes on any *two* of the following : 3×2=6

(a) Red giants

(b) Astronomical coordinate system

(c) Supernova

(d) Expanding universe

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