2016

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

1. *(a)* The coordinates of the two points P and Q are (3, 4, −6) and (1, −2, 3). Find $\overrightarrow{PQ}$. 1

(b) What is the geometrical interpretation of the vector product of two vectors? 1

(c) Find $(\vec{a} \cdot \vec{\nabla})\vec{r}$. 1

(d) Define scalar field and vector field in a region R in space. 1

2. *(a)* If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angel between them, then show that $\sin\frac{\theta}{2}=\frac{1}{2}|\hat{a}-\hat{b}|$. 2

(b) Show that two non-zero vectors

$$\vec{A}=A_1\hat{i}+A_2\hat{j}+A_3\hat{k} \text{ and } \vec{B}=B_1\hat{i}+B_2\hat{j}+B_3\hat{k}$$

are parallel if and only if

$$\frac{A_1}{B_1}=\frac{A_2}{B_2}=\frac{A_3}{B_3}$$ 2

(c) If $\frac{d}{dt}\vec{u}=\vec{w}\times\vec{u}$ and $\frac{d}{dt}\vec{v}=\vec{w}\times\vec{v}$, then show that $\frac{d}{dt}(\vec{u}\times\vec{v})=\vec{w}\times(\vec{u}\times\vec{v})$. 2

(d) If $\vec{A}$ and $\vec{B}$ are irrotational, then prove that $\vec{A}\times\vec{B}$ is solenoidal. 2

3. Answer any *two* questions : 4×2=8

(a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then express $\vec{b}\times\vec{c}$ in terms of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(b) Prove that

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{\nabla}\times\vec{A})-\vec{A}\cdot(\vec{\nabla}\times\vec{B})$$

(c) Find $\vec{\nabla}\cdot\vec{F}$ and $\vec{\nabla}\times\vec{F}$, where

$$\vec{F}=\vec{\nabla}(x^3+y^3+z^3-3xyz)$$

GROUP—B

(Mechanics)

(*Marks* : 40)

4. *(a)* Write two factors on which Coriolis force depends. 1

(b) Give one example of conservative force, which is velocity-dependent. 1

(c) Name the physical quantity which is the cause of rotational motion. Is it a scalar or vector quantity? 1

(d) There is a stick of half wood and half steel. The mass of the wooden portion is less than the steel portion. It is pivoted at the wooden end and force is applied at right angle to its length at the steel end. Next it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular momentum more and why? 1

(e) A thin rod of length L is bent to form a circle and has a mass M. What will be the gravitational potential at the centre? 1

5. *(a)* A non-inertial frame is rotating with angular speed 10 rad/s. A body of mass 20 kg is at a distance 20 cm from the axis of rotation. Find the magnitude of the centrifugal force on the body. 2

(b) Show that, $\vec{F} = (2xy + z^2)\hat{i} + x^2\hat{j} + 2xz\hat{k}$ is a conservative force. 2

(c) An observer in Lab. frame finds the velocities of the two particles A and B of masses m_1 and m_2 as v_1 and v_2. Calculate the velocity of each particle and total momentum of the system in centre of mass frame. 2

(d) State the two theorems of moment of inertia. 2

(e) Gravitational potential is always negative in sign. What conclusion can you draw from this statement? Write the relation between gravitational intensity and gravitational potential. 2

6. Answer any *three* questions : 5×3=15

(a) Find the expression of acceleration in non-inertial frame of reference.

(b) Find the centre of mass of a homogenous semicircular disc.

(c) Show that for compound pendulum, the points of suspension and oscillation are interchangeable.

(d) Derive the expression of gravitational potential at a point outside a spherical shell.

7. Answer any *one* question : 10

(a) (i) What are laboratory and centre of mass frames of reference? 2

(ii) Obtain a relation of scattering angles in these two frames of reference. 8

(b) (i) Calculate the moment of inertia of spherical shell about its diameter. 7

(ii) Two bodies of mass 10 kg and 2 kg are moving with velocities $(2\hat{i}-7\hat{j}+3\hat{k})$ and $(-10\hat{i}+5\hat{j}-3\hat{k})$. Find the velocities of their centres of mass. 3

(c) (i) Derive an expression of deflection of a freely falling body by the Coriolis force due to rotation of earth. 5

(ii) Two bodies of masses M_1 and M_2 are placed at d distance apart. Show that at the position where the gravitational field due to them is zero, the potential is given by

$$V = -\frac{G}{d}[M_1 + M_2 + 2\sqrt{M_1 M_2}]$$ 5

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