2016

PHYSICS

(Major)

Paper : 5.3

(Quantum Mechanics and Astrophysics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Write the answers to the two Groups in separate books

GROUP—A

(Quantum Mechanics)

(*Marks* : 40)

1. Answer any *four* questions as directed : 1×4=4

(a) Mention one experiment which demonstrates the wave nature of matter.

(b) Calculate the energy of a radiowave photon of wavelength $\lambda \approx 10$ m.

(c) Which one of the following is a true statement for photoelectric effect?

(i) Kinetic energy of ejected electrons depends on the intensity of incident light.

(ii) Kinetic energy of ejected electrons depends linearly on the frequency of an incident light.

(iii) Kinetic energy of ejected electrons is always zero.

(iv) It proves wave nature of light.

(Choose the correct option)

(d) Show that de Broglie wavelength of a relativistic particle of mass m moving with velocity v can be approximated as

$$\lambda \approx \frac{h}{mv} - \frac{hv}{2mc^2}$$

(e) The uncertainty of position of a particle of mass m inside a black hole of mass M is about $\Delta x \approx 2GM/c^2$. Here G is Newton's gravitational constant and c is the velocity of light. Calculate the approximate energy of the particle.

2. Answer any *two* questions : 3×2=6

(a) Calculate the Compton wavelength for electron. An X-ray photon of wavelength $\lambda_0 = 1$ Å is incident on a free electron which is initially at rest. The photon is scattered at angle $\theta \simeq 30°$ from the initial direction. Also calculate the final wavelength of the photon. Given, electron mass $m \simeq 9 \cdot 1 \times 10^{-31}$ kg. 1+2=3

(b) The photoelectric work function W for lithium is 2·3 eV. Find the threshold frequency ν_t. If ultraviolet light of wavelength $\lambda = 3000$ Å is incident on a lithium surface, calculate the maximum kinetic energy of the photoelectrons. 1+2=3

(c) Energy of a particle of mass m moving in the gravitational field of a mass M is given by

$$E = \frac{p^2}{2m} - \frac{GMm}{r}$$

Write down the time-independent Schrödinger equation for this system. The wave function of a particle at r is defined as $\psi(r) = f\frac{e^{ikr}}{r}$.

Obtain the probability of finding the particle at r. Here f is a constant complex number. 2+1=3

3. Answer any *four* questions : 5×4=20

(a) The energy distribution of blackbody radiation is given by Planck's law

$$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Show that for long wavelength

$$\rho(\lambda, T) \rightarrow \frac{8\pi kT}{\lambda^4}$$

and for short wavelength

$$\rho(\lambda, T) \rightarrow \frac{8\pi hc}{\lambda^5} \exp\left\{-\frac{hc}{\lambda kT}\right\}$$

What is Planck's quantum hypothesis? Mention one experiment for determining Planck's constant, h. 1½+1½+1+1=5

(b) Show that the ratio of kinetic energy of an alpha particle of mass m_α to that of a proton of mass m_p having same de Broglie wavelength of 1 Å is

$$\frac{KE_\alpha}{KE_p} = \frac{m_p}{m_\alpha}$$

Thermal neutrons in a nuclear reactor have kinetic energy of $3kT/2$, where k is the Boltzmann constant and T is the absolute temperature. What is the energy in eV and de Broglie wavelength of a thermal neutron at room temperature $T \simeq 300$ K? 2+1+2=5

(c) What is the basic difference between Davisson-Germer experiment and G. P. Thomson's experiment on electron diffraction? Show that de Broglie wavelength of an electron of mass m and charge e, accelerated by potential V is

$$\lambda = \frac{h}{\sqrt{2meV}}$$

What is the significance of electron diffraction experiments? Give example of a phenomenon which demonstrates particle nature of light. 1+2+1+1=5

(d) The energy of a particle of mass m moving in a gravitational field of mass M is given by

$$E = \frac{p^2}{2m} - \frac{GMm}{r}$$

Suppose radius of the orbit satisfies the uncertainty principle $rp \simeq \hbar$. Show that energy of the particle becomes

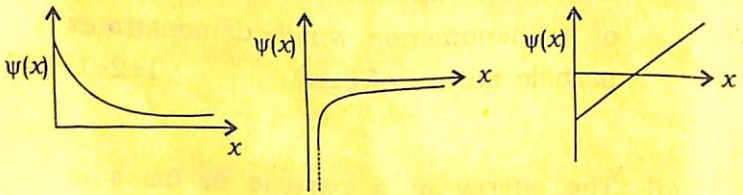
$$E = \frac{\hbar^2}{2mr^2} - \frac{GMm}{r}$$

Also, show that this energy has a minimum at $r = r_0 = (\hbar^2 / GMm^2)$. Obtain the minimum value of energy E_{min}. 1+2+2=5

(e) Briefly discuss G. P. Thomson's experiment of electron diffraction, and its significance for quantum theory. 5

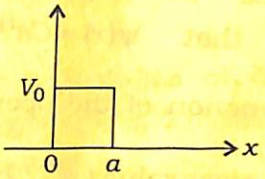
4. Answer any *two* questions : 5×2=10

(a) Which one of the following graphs represents a well-behaved wave function in the range $x \in [0, \infty]$?



Find out the normalization constant A for the wave-function $\psi(x) = A \sin\left(\frac{n\pi x}{2a}\right)$ in the range $-a \leq x \leq a$. 1+4=5

(b) Consider a rectangular potential barrier as shown in the figure



so that $V(x) = \begin{cases} 0, & x < 0 \\ V_0, & 0 < x < a \\ 0, & x > a \end{cases}$

The wave functions in the external regions are given by

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx}, & x < 0 \\ Ce^{ikx}, & x > a \end{cases}$$

(i) Identify incident wave, reflected wave and transmitted wave if the particle comes from the left side.

(ii) Show that the probability current densities are

$$j = \begin{cases} \frac{\hbar k}{m}(|A|^2 - |B|^2), & x < 0 \\ \frac{\hbar k}{m}|C|^2, & x > a \end{cases}$$

(iii) Write down the expression for reflection coefficient R and transmission coefficient T. 1+3+1=5

(c) How do you represent dynamical variables in quantum mechanics? Show that $\psi(x) = Ce^{ipx/\hbar}$ is an eigenfunction of the operator $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$ having eigenvalue $(p^2/2m)$. What is the form of the wave function for a free particle? 1+3+1=5

GROUP—B

(Astrophysics)

(*Marks* : 20)

5. Answer any *three* from the following : 2×3=6

(a) What is the angle of inclination between the ecliptic and the celestial equators? A star has right ascension $\alpha = 6^h51^m$ and another star has $\alpha = 12^h52^m$. Which one of them rises earlier? 1+1=2

(b) Draw a neat diagram of the celestial sphere showing a star in northern hemisphere, the celestial equator, hour angle, observer's meridian and the right ascension of the star. 2

(c) The parallax angle of the star 61 Cygni is 0″.285. Calculate its distance. 2

(d) A star is at a distance of 4 pc. Its apparent magnitude is 2. Calculate its absolute magnitude. 2

(e) Define local sidereal time. If right ascension (α) and declination (δ) of a star are given as ($18^h51^m12^s$, $-05°08'01''$), which hemisphere should be chosen for observing the star? 1+1=2

6. Answer any *two* of the following : 4×2=8

(a) What is the basis of H-D classification of stars? Display the ranges of surface temperatures of *O*-, *B*- and *G*-type stars. In the HR diagram, justify how stars with 'extremely low temperature but large luminosity' and stars with 'extremely high temperature but low luminosity' can exist. 1+1+2=4

(b) Out of pp-chain and CNO cycle, which one dominates the energy production in sun-like stars? What happens to the core of a star once hydrogen burning is exhausted? Discuss how a red giant forms. 1+1+2=4

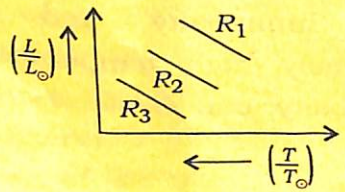
(c) Show that luminosity (L), radius (R) and surface temperature (T) of a star are related as

$$\left(\frac{L}{L_\odot}\right)=\left(\frac{R}{R_\odot}\right)^2\left(\frac{T}{T_\odot}\right)^4$$

where $L_\odot$, $R_\odot$ and $T_\odot$ are corresponding quantities for the sun.

The adjacent figure shows HR diagram with three straight lines across the main sequence, representing stars of radii R_1, R_2 and R_3. Which of the three lines represents the biggest stars?

Show the evolutionary track of a sun-like and a massive, luminous star in the HR diagram. 1+1+2=4



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7. Write short notes on any *two* of the following : 3×2=6

(a) Spectral classification of stars

(b) Celestial coordinates

(c) *PP*-chain and CNO cycle

(d) Trigonometric parallax

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