

2017

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

(Mathematical Methods-II)

(Marks : 35)

1. Answer the following questions : 1×3=3

- (a) Define surface integral of a vector.
- (b) Write down the unit vectors \hat{e}_ρ , \hat{e}_ϕ and \hat{e}_z of a cylindrical coordinate system in terms of \hat{i} , \hat{j} , \hat{k} .
- (c) Give graphical representation of Dirac delta function.

2. If $\vec{V} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, then evaluate $\int_C \vec{V} \cdot d\vec{r}$, where C is a straight line joining $(0, 0, 0)$ and $(1, 1, 1)$. 2

3. Answer any two of the following questions :

5×2=10

- (a) (i) Show that $\Gamma(n+1) = n\Gamma(n)$. 2

- (ii) Find the value of $\Gamma(\frac{1}{2})$. 3

- (b) (i) Show that $\iint_S \vec{r} \cdot \hat{n} dS = 3$, over the surface S of the unit cube bounded by coordinate planes and the planes $x=1, y=1, z=1$. 3

- (ii) From the definition of Dirac delta function, show that

$$\int_{-\infty}^{+\infty} f(x) \delta(x-a) dx = f(a) \quad 2$$

- (c) (i) Express differential operator $\vec{\nabla}$ in terms of orthogonal curvilinear coordinates. 3

- (ii) If $\vec{r} = (u, v)$ represents a surface, then show that the square of the element of arc length on the

surface is

$$dS^2 = E du^2 + 2F du dv + G dv^2$$

where $E = \frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial \vec{r}}{\partial u}$, $F = \frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial \vec{r}}{\partial v}$ and

$$G = \frac{\partial \vec{r}}{\partial v} \cdot \frac{\partial \vec{r}}{\partial v} \quad 2$$

4. Answer any *two* of the following questions :

10×2=20

- (a) (i) If V be the volume bounded by a closed surface S and \vec{A} is a vector function of position with continuous derivatives, then prove that

$$\oiint_S \vec{A} \cdot \hat{n} dS = \iiint_V (\vec{\nabla} \cdot \vec{A}) dV \quad 5$$

- (ii) Prove the following identity : 3

$$\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \iint_S (\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi) \cdot d\vec{S}$$

- (iii) If \hat{n} be the unit outward drawn normal to any closed surface S , find the volume integral

$$\iiint_V \vec{\nabla} \cdot \hat{n} dV \quad 2$$

(b) (i) Express elements of area in orthogonal curvilinear coordinates. 3

(ii) Represent the vector

$$\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$$

in spherical polar coordinate. 5

(iii) Find the volume of a sphere of radius R . 2

(c) (i) Using vector integration, prove the equation of

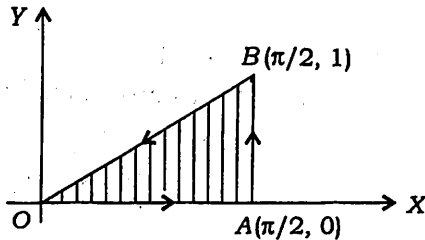
$$\vec{\nabla} \cdot (\rho \vec{v}) + \frac{\partial \rho}{\partial t} = 0$$

where ρ and \vec{v} are the density and velocity of the fluid respectively. 5

(ii) State Green's theorem in the plane and using it evaluate

$$\oint_C (y - \sin x) dx + \cos x dy$$

where C is the triangle of the following figure :



GROUP—B

(Properties of Matter)

(Marks : 25)

5. Answer the following questions : 1×4=4

(a) Write down the relation connecting elastic constants Y , K and η .

(b) A rod is suspended vertically with its upper end rigidly fixed and is twisted at its lower end by means of a couple. For which layer of cylindrical rod shear is maximum?

(c) Why is Poiseuille's equation is not valid for blood flows through veins and capillaries?

(d) Define neutral surface of a beam.

6. Answer the following questions : 2×3=6

(a) Show that for a homogeneous and isotropic material the value of Poisson's ratio lies between -1 and $\frac{1}{2}$, and is equal to $\frac{1}{2}$ only when the material is incompressible.

(b) Write down the Poiseuille's assumptions to determine the rate of flow of liquid through a tube.

- (c) How much work is required to break up a liquid drop of radius R into n equal small drops?

7. Answer any one of the following questions : 5

- (a) (i) Show that simultaneous equal compression and extension at right angles to each other are equivalent to a shear. 3

- (ii) Find the work done in joules in stretching a wire of cross-section 1 sq. mm and length 2 meters through 0.1 mm, if Young's modulus for the material of the wire is 2×10^{12} dynes/cm². 2

- (b) (i) Find an expression for bending moment of a beam. 3

- (ii) A steel rod of length 50 cm, width 2 cm and thickness 1 cm is bent into the form of an arc of radius of curvature 2 m. Calculate the bending moment. Young's modulus of the rod = 2×10^{11} N/m². 2

8. Answer either (a) and (b) or (c) and (d) : 10

Either

(a) Establish the relation between surface energy E and surface tension S of a liquid $E = S - T \frac{dS}{dT}$. 5

(b) (i) Show that angular oscillation of a torsional pendulum is simple harmonic and hence find the time period of the pendulum. 2

(ii) How can moment of inertia of a body be determined with the help of torsional pendulum? 3

Or

(c) Describe with necessary theory the rotating cylinder experiment for the measurement of coefficient of viscosity. 7

(d) An aluminium wire of length 2 m and radius 1 mm is twisted through 90° . Find the angle of shear at the surface, at the axis of the wire and at a point midway between the axis and surface. Calculate the torsional couple, if the modulus of rigidity is $5 \times 10^{10} \text{ N/m}^2$. 3
