2017

PHYSICS

(Major)

Paper : 3.1

(Mathematical Methods–III and Electrostatics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Answer the following questions : $1\times3=3$

(a) What do you mean by an idempotent matrix?

(b) Show that

$$A=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$$

is a self-adjoint matrix.

(c) Is $S = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ a symmetric matrix?

2. Show that

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$$

is a nilpotent matrix. 2

3. Answer any *two* of the following questions : 5×2=10

(a) (i) Show that every square matrix can be expressed uniquely as the sum of a Hermitian and a skew-Hermitian matrix. 2

(ii) For the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$, verify the theorem

$$A\,(\text{adj } A) = (\text{adj } A)\,A = |A|\,I$$ 3

(b) (i) Show that the trace of a product of two matrices is independent of the order of multiplication. 2

(ii) Determine the eigenvalues of the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. 3

(c) (i) Prove that the product of a singular matrix with its adjoint is the null matrix. 2

(ii) What is the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -3 & -6 & -9 \end{pmatrix} ?$$ 3

4. Answer *either* (a) and (b) *or* (c) and (d) :

5×2=10

(a) (i) For three Pauli spin matrices $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, show that $\sigma_1 \sigma_2 = i\sigma_3$. 2

(ii) Prove that the modulus of each eigenvalue of a unitary matrix is unitary. 3

(b) Diagonalize the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$ 5

(c) State Cayley-Hamilton theorem. Obtain the eigenvalue equation of the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

and verify the Cayley-Hamilton theorem. 5

(d) Using matrix, find the transformation equation for a 90 degree rotation about the z-axis of Cartesian coordinate system. 5

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct answer/Answer the following questions : 1×4=4

(a) Potential at a large distance from a quadrupole is proportional to

(i) $\frac{1}{r}$ (ii) $\frac{1}{r^2}$

(iii) $\frac{1}{r^3}$ (iv) $\frac{1}{r^4}$

(b) How much electric energy is stored by a solid sphere of radius R and total charge Q?

(c) Electric field vector is

(i) rotational

(ii) irrotational

(d) Write Gauss' law of electrostatics in differential form.

6. Answer the following questions : 2×3=6

(a) Draw a schematic diagram of an electric octopole.

(b) Two charges 5 microcoulomb each are at the points (0, 3, 0) and (3, 0, 0) respectively. Find the electric intensity at (0, 0, 5).

(c) The electric field due to a short dipole at a point distant 1 cm from it on its perpendicular bisector is $1{\cdot}5 \times 10^{-11}$ volt/m. Find the dipole moment.

7. Using integral form of Gauss' law in electrostatics, find the electric field and potential at a distance z above the centre of a circular loop of radius R that carries a uniform line charge λ. 5

Or

Find an expression for the torque experienced by an electric dipole in external electric field. Hence show that the work done in rotating the dipole from an initial position θ_1 to θ_2 is $W = -pE(\cos\theta_2 - \cos\theta_1)$. 3+2=5

8. Answer any *two* questions : 10×2=20

(a) (i) Using Gauss' law, find the electric field outside a uniformly charged solid sphere of radius R and total charge q. 3

(ii) Compute the curl and divergence of electric field given by

$$\vec{E}(x, y, z) = (x^2 + z^2 + 5)\hat{i} + (x^2 + y^2 - 9z)\hat{j} + (y^2 + z^2)\hat{k}$$ 2

(iii) Establish the Clausius-Mosotti equation for a linear dielectric material. 5

(b) (i) Define electric susceptibility, permeability and dielectric constant of material. 3

(ii) Using Laplace's equation, obtain an expression for potential inside a spherical capacitor of two concentric spheres having inner radius a and outer radius b. 3

(iii) State and prove uniqueness theorem. 4

(c) (i) An isotropic dielectric is placed in an uniform electrostatic field E. Show that the field inside a spherical cavity in this direction is

$$E_i = E + \frac{P}{3\,\varepsilon_0}$$

where P is the polarization. 5

(ii) Write Poisson's equation. Solve Laplace's equation to find the potential at a distance r from the axis of an infinitely long conducting cylinder of radius R charged with a surface density σ. 1+4=5

(d) (i) A point charge is situated near an infinite plane earthed conductor. Apply the method of electrical image to calculate—

1. the surface charge density induced on the plane;
2. the force between the plane and the charge. 4+3=7

(ii) An electron is at a distance 10 Å from an infinite plane conductor. Calculate the force experienced by the electron and the work done in moving it to infinite distance away from the conductor. 3

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