# 2017

## PHYSICS

## ( Major )

Paper : 4.1

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## GROUP—A

## ( Mathematical Methods—IV )

( *Marks* : 40 )

**1.** Answer any *four* of the following questions : 1×4=4

*(a)* Write a second-order, second-degree differential equation.

*(b)* If $P_n(x)$ and $Q_n(x)$ are two independent solutions of Legendre equation, then write the most general solution of the Legendre equation.

(c) What is meant by mean deviation?

(d) Write the Hermite equation that can be solved by the power series method, where there is no singularity in the finite plane.

(e) What is total probability?

(f) Give an example where solution of Legendre differential equation is used.

**2.** Answer any *three* of the following questions : 2×3=6

(a) Prove that $P_n^m(-x) = (-1)^{m+n} P_n^m(x)$.

(b) Check whether Frobenius method can be applied or not to the following equation :

$$2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

(c) Calculate the probability of obtaining 4 heads in 6 tosses using an unbiased coin.

(d) Explain briefly Gaussian distribution.

(e) Prove the following recurrence relation :

$$2x H_n(x) = 2n H_{n-1}(x) + H_{n+1}(x)$$

(f) Show that $P_n(1) = 1$.

**3.** Answer any *two* of the following questions : 5×2=10

(a) Prove that

$$\int_{-1}^{+1} x P_n(x) P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1}$$

(b) Find the indicial equation of the following Hermite equation :

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2xy = 0$$

(c) Find the probability that in 100 tosses of a coin, we get between 45 and 55 heads.

(d) If the solution $y(x)$ of Hermite's differential equation is written as

$$y(x) = \sum_{r=0}^{\infty} a_r x^{k+r}$$

then show that the allowed values of $k$ are zero and one only.

**4.** Answer any *two* of the following questions : 10×2=20

*(a)* *(i)* Show that

$$\int_{-1}^{+1} P_n(x)\, P_m(x)\, dx = 0$$

where $P_n(x)$ and $P_m(x)$ are solutions of the Legendre differential equation. 6

*(ii)* Prove the following recurrence relation : 4

$$n P_n = (2n-1)\, x\, P_{n-1} - (n-1)\, P_{n-2}$$

*(b)* *(i)* Show that the generating function for Hermite polynomial $H_n(x)$ for integral $n$ and real values of $n$ is given by

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$$ 6

*(ii)* Prove the following recurrence relation : 4

$$2n\, H_{n-1}(x) = H_n'(x)$$

*(c)* *(i)* What do you mean by variance? 1

*(ii)* What is a normal curve and under what condition a normal curve becomes a normal distribution? 1+2=3



( Continued )

(iii) Define standard deviation. In a project, the height of 5 cats are found as 60 mm, 47 mm, 30 mm, 43 mm and 20 mm. Find the standard deviation. 1+4=5

(iv) Define mutually exclusive events. 1

(d) (i) Solve by power series method, the following equation : 5

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$$

(ii) Identify the singular points of the Legendre differential equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-n(n+1)y=0;$$

$n$ = constant

Obtain the general series solution near $n=0$. 5

GROUP—B

**( Introduction to Computer and Computer Programming )**

( *Marks* : 20 )

**5.** Answer any *three* of the following questions : 1×3=3

*(a)* Define a flowchart.

*(b)* What is a string?

*(c)* What is word length of the computer?

*(d)* What is the standard input stream in C / C++ / FORTRAN to access the keyboard?

**6.** Answer any *one* of the following questions : 2

*(a)* What are the differences between machine language and high-level language?

*(b)* What are different types of logical operations in C / C++ / FORTRAN?

*(c)* Give a typical declaration for an array in C / C++ / FORTRAN.

**7.** Answer any *one* of the following questions : 5

*(a)* Write an algorithm to find whether a given number is odd or even.

*(b)* Write a program in C / C++ / FORTRAN to find the compound interest.

**8.** Answer any *one* of the following questions : 10

*(a)* Write the algorithm and draw the flowchart to find the roots of a quadratic equation.

*(b)* Write the algorithm and the program in C / C++ / FORTRAN to find the sum and average of first *n* natural numbers.

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