2017

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 30)

1. Answer the following questions : $e^{i\theta} = \cos\theta + i\sin\theta$ $1\times4=4$

(a) Give the Euler's formula.

(b) For the complex number $z = 3 - 4i$, find z^4, given that $\tan^{-1}\frac{4}{3} = 53\cdot13°$.

(c) Plot the number $e^{(1+\frac{\pi}{4}i)}$.

(d) What is Argand plane or complex plane?

2. *(a)* State De Moivre's theorem. 2

(b) Using De Moivre's theorem, show that $e^{in\theta} = \cos n\theta + i \sin n\theta$. 2

3. *(a)* Examine whether the function $f(z) = (x + iy)^3$ is an analytic function or not. 3

(b) Prove that $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$. 2

Or

Check the analyticity of the function $f(z) = \ln z$ and hence find its derivative. 5

4. *(a)* State and prove Cauchy's integral theorem. 6

(b) Using Cauchy's integral formula, evaluate $\oint \frac{z-1}{z^2+1} dz$ around the contours—

(i) $|z - i| = 1$

(ii) $|z| = 2$ 2+2=4

5. Answer either *(a)* and *(b)* or *(c)* and *(d)* :

(a) State and prove Taylor's theorem. 5

(b) Evaluate $\oint_c \frac{dz}{z}$, where c is a circle of unit radius. 2

(c) Obtain the residue of the function $f(z) = \frac{e^{iz}}{x^2 + a^2}$ at $z = ia$. 2

(d) Evaluate the given definite integral by the calculus of residues $\int_{-\infty}^{+\infty} \frac{dx}{(x^2 + a^2)^2}$. 5

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

6. Answer the following questions : 1×4=4

(a) What do you mean by constraints?

(b) What is a central force?

(c) What do you mean by a cyclic coordinate?

(d) What is areal velocity of a particle?

7. Answer any *three* of the following questions : 2×3=6

(a) What are generalized coordinates?

(b) Define virtual displacement and discuss its significance.

(c) Distinguish between Lagrangian and Hamiltonian formalisms.

(d) Define reduced mass in the context of two-body central force problem.

(e) Show that angular momentum is a constant of central force motion.

8. (a) State and establish d'Alembert's principle. 1+3=4

Or

Define Hamiltonian of a system. Under what conditions is it equivalent to energy? 1+3=4

(b) Show that a two-body central force problem can be reduced to one-body problem. 4

(c) Establish the Hamiltonian and hence the equation of motion of a simple pendulum. 5

Or

Establish the differential equation for the orbit under central force. 5

9. Find the equation of motion of a compound pendulum by using Lagrange's equation. Hence find an expression for period of oscillation for the pendulum. 5+2=7

Or

Derive Lagrange's equation of motion from Hamilton's principle for a conservative system. 7

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